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The matching equivalent classes of $P_4 \cup I_6 \cup T(1,1,n)$

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Abstract: **Completely characterize the matching equivalent classes of $P4 \cup I_6 \cup T(1,1,n)$, by the property of graph's matching polynomial and its maxinum roots.**

Keywords: **Matching polynomial, Matching equivalence, Matching uniqueness, The maxinum real roots.**

I. INTRODUCTION

All graphs considered in this paper are undirected and simple (i.e., loops and multiple edges are not allowed). Let $G=(V(G),E(G))$ be a graph with vertex set $V(G)=\{v_1,v_2,\cdots,v_n\}$ and edge set $E(G)=\{e_1,e_2,\ \cdots,e_m\}$, where $|V(G)|=n$ is the order and $|E(G)|=m|$ is the size of *G*. A spanning subgraph *H* is called a matching of *G*, if every connected component of *H* is isolated edge or isolated vertex. *k*-matching of *G* is a matching with *k* edges. In [1] E. J. Farrell denote the matching polynomial as

$$\mu(G,x)=\sum_{k\ge 0}(-1)^k p(G,k)x^{n-2k},$$

where $p(G,k)$ is the number of *k*-matchings of *G*.

Two graphs *G* and *H* are called matching-equivalent if $\mu(G,x)=\mu(H,x)$, and denoted by $G \sim H$. The disjoint union of two graphs *G* and *H*, denoted by $G \bigcup H$, is the graph with vertex set $V(G)\cup V(H)$ and edge set $E(G)\cup E(H)$. *kG* denotes the disjoint union of *k* copies of *G*. Let *M*(*G*) be the largest matching root of $\mu(G,x)$. We denote by P_n $(n\ge 1)$, C_n $(n\ge 3)$ a path and a cycle of order *n*, respectively. A graph $D_{m,n}(m\ge 3,\ \ n\ge 1)$ is defined as the graph obtained by identifying one end of the path P_{n+1} with a vertex of the cycle C_m. By $T(a,b,c)$ denote the tree which has one 3-degree vertex *u* and three 1-degree vertices v_1,v_2,v_3 and the distance between *u* and v_1,v_2,v_3 are a,b,c, respectively. Let P_{n-2} be a path with vertices sequence 1, 2, …, *n* −2, $I_n(n\ge 6)$ denotes the tree obtained by adding pendant edges at vertices 2 and *n* – 3 of P_{n-2}, respectively. For a graph *G*, let G^c be the complement of *G*. The classes of matching equivalent graphs determined by *G* under ~ is denoted by [*G*]. For undefined terminology and notations we refer to references [2]. In this paper, we Completely characterize the matching equivalent classes of $P_4 \bigcup I_6 \bigcup T(1,1,n)$.

II. BASIC LEMMA

Lemma 2.1 ([1]): *Let G be a graph with k components* $G_1,G_2,....,G_k$. *Then* $\mu(G,x)=\prod_{i=1}^{k}\mu(G_i,x)$.

Lemma 2.2 ([1]): *Let* $e=uv\in E(G)$. *Then* $\mu(G,x)=\mu(G-e,x)-\mu(G-\{u,v\},x)$.

Lemma 2.3 ([1]): *Let G be a connected graph and* $u\in V(G)$, $e\in E(G)$. *Then M(G) is a single root of* $\mu(G,x)$ *and* $M(G)>M(G-u)$, $M(G)>M(G-e)$.

Lemma 2.4 ([3]): *Let G be a connected graph. Then*

(1) $M(G) < 2$ *if and only if* $G \in \Omega_1 = \{K_1, P_n, C_n, T(1,1,n), T(1,2,i)(2 \le i \le 4), D_{3,1}\}$;

(2) $M(G) = 2$ *if and only if* $G \in \Omega_2 = \{K_{1,4}, T(2,2,2), T(1,3,3), T(1,2,5), I_n, D_{3,2}, D_{4,1}\}$.

Lemma 2.5 ([4]): *Let* $M(G) < 2$. *Then graph G is matching uniquely if and only if*

$$G = kK_1 \cup m_2P_2 \cup m_3P_3 \cup [\bigcup_{i\ge 2} m_{2i}P_{2i}] \cup [\bigcup_{j\ge 3} n_jC_j] \cup dD_{3,1} \cup eT(1,2,3) \cup fT(1,2,4),$$

where $kn_j = m_in_{i+1} = m_2d = m_3d = n_3e = n_{15}e = n_3n_5f = n_5n_9f = 0$ *and* k, m_i, n_j, d, e, f *are non-negative integer.*

Lemma 2.6 ([5]): (1) $P_{2m+1} \sim P_m \cup C_{m+1}\ (m \ge 2)$ (2) $T(1,1,n) \sim K_1 \cup C_{n+2}$ (3) $T(1,2,2) \sim P_2 \cup D_{3,1}$ (4) $K_1 \cup C_6 \sim P_3 \cup D_{3,1}$ (5) $K_1 \cup C_9 \sim C_3 \cup T(1,2,3)$ (6) $K_1 \cup C_{15} \sim C_3 \cup C_5 \cup T(1,2,4)$

Lemma 2.7 ([6]): (1) $D_{3,2} \sim D_{4,1}$ (2) $K_1 \cup D_{3,2} \sim I_6$ (3) $T(2,2,2) \sim P_2 \cup D_{3,2}$ (4) $T(1,3,3) \sim P_3 \cup D_{3,2}$ (5) $T(1,2,5) \sim P_4 \cup D_{3,2}$ (6) $K_1 \cup I_6 \sim P_2 \cup K_{1,4}$ (7) $P_{m-4} \cup I_n \sim P_{n-4} \cup I_m\ (m, n \ge 6)$ (8) $I_{2m-3} \sim I_m \cup C_{m-3}\ (m \ge 6)$

Lemma 2.8 ([5]): $G \sim H$ if and only if $G^c \sim H^c$.

III. MAIN RESULTS

Theorem 3.1 *Let* $[P_4 \cup I_6 \cup T(1,1,n)]$ *be The classes of matching equivalent graphs of* $P_4 \cup I_6 \cup T(1,1,n)$. *Then*

(1) *If* $n \ne 1,3,4,7,13$, $[P_4 \cup I_6 \cup T(1,1,n)] = \{P_4 \cup P_2 \cup K_{1,4} \cup C_{n+2},\ K_1 \cup T(1,2,5) \cup T(1,1,n), 2K_1 \cup T(1,2,5) \cup C_{n+2}, P_4 \cup I_6 \cup K_1 \cup C_{n+2}, P_2 \cup I_8 \cup T(1,1,n), P_2 \cup I_8 \cup K_1 \cup C_{n+2}, P_4 \cup K_1 \cup D_{3,2} \cup T(1,1,n), P_4 \cup 2K_1 \cup D_{3,2} \cup C_{n+2}, P_4 \cup K_1 \cup D_{4,1} \cup T(1,1,n), P_4 \cup 2K_1 \cup D_{4,1} \cup C_{n+2}\}$;

(2) *If* $n = 1$, $[P_4 \cup I_6 \cup T(1,1,1)] = \{P_4 \cup K_{1,4} \cup P_2 \cup C_3, P_4 \cup K_{1,4} \cup P_5, P_4 \cup I_6 \cup K_1 \cup C_3,\ P_2 \cup I_8 \cup T(1,1,1), P_2 \cup I_8 \cup K_1 \cup C_3, I_8 \cup K_1 \cup P_5, P_4 \cup I_9 \cup K_1\}$;

(3) *If* $n = 3$, $[P_4 \cup I_6 \cup T(1,1,3)] = \{P_4 \cup K_{1,4} \cup P_2 \cup C_5, P_2 \cup K_{1,4} \cup P_9,\ P_4 \cup I_6 \cup K_1 \cup C_5, I_6 \cup K_1 \cup P_9,\ P_2 \cup I_8 \cup T(1,1,3), P_2 \cup I_8 \cup K_1 \cup C_5, P_2 \cup I_{13} \cup K_1,\ P_4 \cup K_1 \cup D_{3,2} \cup T(1,1,3), P_4 \cup 2K_1 \cup D_{3,2} \cup C_5, 2K_1 \cup D_{3,2} \cup P_9,\ P_4 \cup K_1 \cup D_{4,1} \cup T(1,1,3), P_4 \cup 2K_1 \cup D_{4,1} \cup C_5, 2K_1 \cup D_{4,1} \cup P_9\}$;

(4) *If* $n = 4$, $[P_4 \cup I_6 \cup T(1,1,4)] = \{T(1,2,5) \cup K_1 \cup T(1,1,4), T(1,2,5) \cup 2K_1 \cup C_6, T(1,2,5) \cup K_1 \cup P_3 \cup D_{3,1}, P_4 \cup I_6 \cup K_1 \cup C_6, P_4 \cup P_3 \cup I_6 \cup D_{3,1}, P_2 \cup I_8 \cup T(1,1,4), P_2 \cup I_8 \cup K_1 \cup C_6, P_2 \cup I_8 \cup P_3 \cup D_{3,1},\ P_3 \cup I_8 \cup T(1,2,2), P_4 \cup K_1 \cup D_{3,2} \cup T(1,1,4), P_4 \cup 2K_1 \cup D_{3,2} \cup C_6, P_4 \cup P_3 \cup K_1 \cup D_{3,2} \cup D_{3,1},\ P_4 \cup K_1 \cup D_{4,1} \cup T(1,1,4), P_4 \cup 2K_1 \cup D_{4,1} \cup C_6, P_4 \cup P_3 \cup K_1 \cup D_{4,1} \cup D_{3,1}\}$;

(5) *If* $n = 7$, $[P_4 \cup I_6 \cup T(1,1,7)] = \{T(1,2,5) \cup K_1 \cup T(1,1,7), T(1,2,5) \cup 2K_1 \cup C_9, T(1,2,5) \cup K_1 \cup C_3 \cup T(1,2,3), P_4 \cup I_6 \cup K_1 \cup C_9, P_4 \cup I_6 \cup C_3 \cup T(1,2,3), P_2 \cup I_8 \cup T(1,1,7), P_2 \cup I_8 \cup K_1 \cup C_9, P_2 \cup I_8 \cup C_3 \cup T(1,2,3), P_5 \cup I_8 \cup T(1,2,3),\ K_1 \cup P_4 \cup D_{3,2} \cup T(1,1,7),\ 2K_1 \cup P_4 \cup D_{3,2} \cup C_9, K_1 \cup P_4 \cup D_{3,2} \cup C_3 \cup T(1,2,3),\ K_1 \cup P_4$

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$\cup D_{4,1}\cup T(1,1,7)$, $2K_1\cup P_4\cup D_{4,1}\cup C_9, K_1\cup P_4\cup D_{4,1}\cup C_3\cup T(1,2,3)$ };

(6) *If* $n=13$, $[P_4\cup I_6\cup T(1,1,13)]=\{K_1\cup T(1,2,5)\cup T(1,1,13), 2K_1\cup T(1,2,5)\cup C_{15}, K_1\cup T(1,2,5)\cup C_3\cup C_5$ $\cup T(1,2,4)$, $P_4\cup I_6\cup K_1\cup C_{15}, P_4\cup I_6\cup C_3\cup C_5\cup T(1,2,4), C_3\cup P_9\cup I_6\cup T(1,2,4), P_2\cup I_8\cup T(1,1,13)$, $P_2\cup I_8\cup K_1\cup C_{15}$, $P_2\cup I_8\cup C_3\cup C_5\cup T(1,2,4), P_5\cup I_8\cup C_5\cup T(1,2,4)$, $P_2\cup I_{13}\cup C_3\cup T(1,2,4), P_5\cup I_{13}\cup T(1,2,4)$, $K_1\cup P_4\cup D_{3,2}\cup T(1,1,13), 2K_1\cup P_4\cup D_{3,2}\cup C_{15}$, $K_1\cup P_4\cup D_{3,2}\cup C_3\cup C_5\cup T(1,2,4)$, $K_1\cup C_3\cup P_9\cup D_{3,2}\cup T(1,2,4)$, $K_1\cup P_4\cup D_{4,1}\cup T(1,1,13), 2K_1\cup P_4\cup D_{4,1}\cup C_{15}$, $K_1\cup P_4\cup D_{4,1}\cup C_3\cup C_5\cup T(1,2,4)$, $K_1\cup C_3\cup P_9\cup D_{4,1}\cup T(1,2,4)$ }.

Proof: Let $H\sim P_4\cup I_6\cup T(1,1,n)$, then $M(H)=M(P_4\cup I_6\cup T(1,1,n))$, By Lemma 2.4, we obtain $M(H)=M(I_6)=2$. There must be a connected component H_1 in H belonging to the set Ω_2. Let $H= H_1\cup H_2$. Thus, we distinguish with the following cases according to $M(I_6)$.

Case 1: If $H_1=K_{1,4}$, then $P_4\cup I_6\cup T(1,1,n)\sim K_{1,4}\cup H_2$. By $K_{1,4}\cup H_2\sim P_4\cup I_6\cup T(1,1,n)\sim P_3\cup I_6\cup K_1\cup C_{n+2}\sim P_2\cup P_4\cup K_{1,4}\cup C_{n+2}$, we have $H_2\sim P_2\cup P_4\cup C_{n+2}$.

Subcase 1.1: If $n\neq 1,3$, then $H_2\sim P_2\cup P_4\cup C_{n+2}$;

Subcase 1.2: If n=1, by Lemma 2.6, we have $H_2\sim P_2\cup P_4\cup C_3\sim P_4\cup P_5$;

Subcase 1.3: If n=2, by Lemma 2.6, we have $H_2\sim P_2\cup P_4\cup C_5\sim P_2\cup P_9$.

Case 2: If $H_1=T(2,2,2)$, then $P_4\cup I_6\cup T(1,1,n)\sim T(2,2,2)\cup H_2$. By $K_1\cup P_4\cup I_6\cup T(1,1,n)\sim K_1\cup T(2,2,2)\cup H_2\sim K_1\cup P_2\cup D_{3,2}\cup H_2\sim P_2\cup I_6\cup H_2$, we have $P_2\cup H_2\sim K_1\cup P_4\cup T(1,1,n)\sim 2K_1\cup P_4\cup C_{n+2}$. By Lemma 2.5 and Lemma 2.6, this case does not exist.

Case 3: If $H_1=T(1,3,3)$, then $P_4\cup I_6\cup T(1,1,n)\sim T(1,3,3)\cup H_2$. By $K_1\cup P_4\cup I_6\cup T(1,1,n)\sim K_1\cup T(1,3,3)\cup H_2\sim K_1\cup P_3\cup D_{3,2}\cup H_2\sim P_3\cup I_6\cup H_2$, we have $P_3\cup H_2\sim K_1\cup P_4\cup T(1,1,n)\sim 2K_1\cup P_4\cup C_{n+2}$. By Lemma 2.5 and Lemma 2.6, this case does not exist.

Case 4: If $H_1=T(1,2,5)$, then $P_4\cup I_6\cup T(1,1,n)\sim T(1,2,5)\cup H_2$. By $K_1\cup P_4\cup I_6\cup T(1,1,n)\sim K_1\cup T(1,2,5)\cup H_2\sim K_1\cup P_4\cup D_{3,2}\cup H_2\sim P_4\cup I_6\cup H_2$.

Subcase 4.1: If $n\neq 4,7,13$, By Lemma 2.6, we have $H_2\sim K_1\cup T(1,1,n)\sim 2K_1\cup C_{n+2}$;

Subcase 4.2: If $n=4$, By Lemma 2.6, we have $H_2\sim K_1\cup T(1,1,4)\sim 2K_1\cup C_6\sim K_1\cup P_3\cup D_{3,1}$;

Subcase 4.3: If $n=7$, By Lemma 2.6, we have $H_2\sim K_1\cup T(1,1,7)\sim 2K_1\cup C_9\sim K_1\cup C_3\cup T(1,2,3)$;

Subcase 4.4: If $n=13$, By Lemma 2.6, we have $H_2\sim K_1\cup T(1,1,13)\sim 2K_1\cup C_{15}\sim K_1\cup C_3\cup C_5\cup T(1,2,4)$.

Case 5: If $H_1=I_m$, then $P_4\cup I_6\cup T(1,1,n)\sim I_m\cup H_2$.

Subcase 5.1: If $n\neq 1,3,4,7,13$, By Lemma 2.6 and Lemma 2.7, we have $I_m\cup H_2\sim P_4\cup I_6\cup T(1,1,n)\sim P_4\cup I_6\cup K_1\cup C_{n+2}\sim P_2\cup I_8\cup T(1,1,n)\sim P_2\cup I_8\cup K_1\cup C_{n+2}$. So, if $m=6$, we have $H_2\sim P_4\cup T(1,1,n)\sim P_4\cup K_1\cup C_{n+2}$; if $m=8$, we have $H_2\sim P_2\cup T(1,1,n)\sim P_2\cup K_1\cup C_{n+2}$.

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Subcase 5.2: If n=1, By Lemma 2.6 and Lemma 2.7, we have $I_m \cup H_2 \sim P_4 \cup I_6 \cup T(1,1,1) \sim P_4 \cup I_6 \cup K_1 \cup C_3 \sim P_2 \cup I_8 \cup T(1,1,1) \sim P_2 \cup I_8 \cup K_1 \cup C_3 \sim K_1 \cup P_5 \cup I_8 \sim K_1 \cup P_4 \cup I_9$.

So, if $m=6$, we have $H_2 \sim P_4 \cup T(1,1,1) \sim P_4 \cup K_1 \cup C_3$; if $m=8$, $H_2 \sim P_2 \cup T(1,1,1) \sim P_2 \cup K_1 \cup C_3 \sim K_1 \cup P_5$; if $m=9$, $H_2 \sim K_1 \cup P_4$.

Subcase 5.3: If n=3, By Lemma 2.6 and Lemma 2.7, we have $I_m \cup H_2 \sim P_4 \cup I_6 \cup T(1,1,3) \sim P_4 \cup I_6 \cup K_1 \cup C_5 \sim K_1 \cup P_9 \cup I_6 \sim P_2 \cup I_8 \cup T(1,1,3) \sim P_2 \cup I_8 \cup K_1 \cup C_5 \sim K_1 \cup P_2 \cup I_{13}$. So, if $m=6$, we have $H_2 \sim P_4 \cup T(1,1,3) \sim P_4 \cup K_1 \cup C_5 \sim K_1 \cup P_9$; if $m=8$, we have $H_2 \sim P_2 \cup T(1,1,3) \sim P_2 \cup K_1 \cup C_5$; if $m=13$, we have $H_2 \sim K_1 \cup P_2$.

Subcase 5.4: If n=4, By Lemma 2.6 and Lemma 2.7, we have $I_m \cup H_2 \sim P_4 \cup I_6 \cup T(1,1,4) \sim P_4 \cup I_6 \cup K_1 \cup C_6 \sim P_3 \cup P_4 \cup I_6 \cup D_{3,1} \sim P_2 \cup I_8 \cup T(1,1,4) \sim P_2 \cup I_8 \cup K_1 \cup C_6 \sim P_2 \cup P_3 \cup I_8 \cup D_{3,1} \sim I_8 \cup P_3 \cup T(1,2,2)$. So, if $m=6$, we have $H_2 \sim P_4 \cup T(1,1,4) \sim P_4 \cup K_1 \cup C_6 \sim P_3 \cup P_4 \cup D_{3,1}$; if $m=8$, we have $H_2 \sim P_2 \cup T(1,1,4) \sim P_2 \cup K_1 \cup C_6 \sim P_2 \cup P_3 \cup D_{3,1} \sim P_3 \cup T(1,2,2)$.

Subcase 5.5: If n=7, By Lemma 2.6 and Lemma 2.7, we have $I_m \cup H_2 \sim P_4 \cup I_6 \cup T(1,1,7) \sim P_4 \cup I_6 \cup K_1 \cup C_9 \sim P_4 \cup I_6 \cup C_3 \cup T(1,2,3) \sim P_2 \cup I_8 \cup T(1,1,7) \sim P_2 \cup I_8 \cup K_1 \cup C_9 \sim P_2 \cup I_8 \cup C_3\ P_2 \cup I_8 \cup \sim P_5 \cup I_8 \cup T(1,\ 2,3)$. So, if $m=6$, we have $H_2 \sim P_4 \cup T(1,1,7) \sim P_4 \cup K_1 \cup C_9 \sim P_4 \cup C_3 \cup T(1,2,3)$; if $m=8$, we have $H_2 \sim P_2 \cup T(1,1,7) \sim P_2 \cup K_1 \cup C_9 \sim P_2 \cup C_3 \cup T(1,2,3) \sim P_5 \cup (1,2,3)$.

Subcase 5.6: If n=13, By Lemma 2.6 and Lemma 2.7, we have $I_m \cup H_2 \sim P_4 \cup I_6 \cup T(1,1,13) \sim P_4 \cup I_6 \cup K_1 \cup C_{15} \sim P_4 \cup I_6 \cup C_3 \cup C_5 \cup T(1,2,4) \sim C_3 \cup P_9 \cup I_6 \cup T(1,2,4) \sim P_2 \cup I_8 \cup T(1,1,13) \sim P_2 \cup I_8 \cup K_1 \cup C_{15} \sim P_2 \cup I_8 \cup C_3 \cup C_5 \cup T(1,2,4) \sim P_2 \cup I_{13} \cup C_3 \cup T(1,2,4) \sim P_5 \cup I_{13} \cup T(1,2,4)$. So, if $m=6$, we have $H_2 \sim P_4 \cup T(1,1,13) \sim P_4 \cup K_1 \cup C_{15} \sim P_4 \cup C_3 \cup C_5 \cup T(1,2,4) \sim C_3 \cup P_9 \cup T(1,2,4)$; if $m=8$, we have $H_2 \sim P_2 \cup T(1,1,13) \sim P_2 \cup K_1 \cup C_{15} \sim P_2 \cup C_3 \cup C_5 \cup T(1,2,4) \sim P_5 \cup C_5 \cup T(1,2,4)$; if $m=13$, we have $H_2 \sim P_2 \cup C_3 \cup T(1,2,4) \sim P_5 \cup T(1,2,4)$.

Case 6: If $H_1 = D_{3,2}$, then $P_4 \cup I_6 \cup T(1,1,n) \sim D_{3,2} \cup H_2$. By Lemma 2.7, we have $K_1 \cup P_4 \cup I_6 \cup T(1,1,n) \sim K_1 \cup D_{3,2} \cup H_2 \sim I_6 \cup H_2$.

Subcase 6.1: If $n \neq 3,4,7,13$, By Lemma 2.6, we have $H_2 \sim K_1 \cup P_4 \cup T(1,1,n) \sim 2K_1 \cup P_4 \cup C_{n+2}$;

Subcase 6.2: If $n=3$, By Lemma 2.6, we have $H_2 \sim K_1 \cup P_4 \cup T(1,1,3) \sim 2K_1 \cup P_4 \cup C_5 \sim 2K_1 \cup P_9$;

Subcase 6.3: If $n=4$, By Lemma 2.6, we have $H_2 \sim K_1 \cup P_4 \cup T(1,1,4) \sim 2K_1 \cup P_4 \cup C_6 \sim K_1 \cup P_3 \cup P_4 \cup D_{3,1}$;

Subcase 6.4: If $n=7$, By Lemma 2.6, we have $H_2 \sim K_1 \cup P_4 \cup T(1,1,7) \sim 2K_1 \cup P_4 \cup C_9 \sim K_1 \cup P_4 \cup C_3 \cup T(1,2,3)$;

Subcase 6.5: If $n=13$, By Lemma 2.6, we have $H_2 \sim K_1 \cup P_4 \cup T(1,1,13) \sim 2K_1 \cup P_4 \cup C_{15} \sim K_1 \cup P_4 \cup C_3 \cup C_5 \cup T(1,2,4) \sim K_1 \cup C_3 \cup P_9 \cup T(1,2,4)$.

Case 7: If $H_1 = D_{4,1}$. By $D_{3,2} \sim D_{4,1}$, it is similar to Case 6. The proof is omitted.

The proof of Theorem 3.1 is complete.

By Theorem 3.1 and Lemma 2.8, we have Theorem 3.2:

Theorem 3.2 *Let* $[(P_4 \cup I_6 \cup T(1,1,n))^c]$ *be The classes of matching equivalent graphs of* $(P_4 \cup I_6 \cup T(1,1,n))^c$. *Then*

(1) *If* $n \neq 1,3,4,7,13$, $[(P_4 \cup I_6 \cup T(1,1,n))^c] = \{(P_4 \cup P_2 \cup K_{1,4} \cup C_{n+2})^c$, $(K_1 \cup T(1,2,5) \cup T(1,1,n))^c, (2K_1 \cup T(1,2,5) \cup C_{n+2})^c, (P_4 \cup I_6 \cup K_1 \cup C_{n+2})^c$, $(P_2 \cup I_8 \cup T(1,1,n))^c, (P_2 \cup I_8 \cup K_1 \cup C_{n+2})^c, (P_4 \cup K_1 \cup D_{3,2} \cup T(1,1,\ n))^c$, $(P_4 \cup 2K_1 \cup D_{3,2} \cup$

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$C_{n+2})^c$, $(P_4 \cup K_1 \cup D_{4,1} \cup T(1,1,n))^c, (P_4 \cup 2K_1 \cup D_{4,1} \cup C_{n+2})^c\}$;

(2) *If* $n=1$, $[(P_4 \cup I_6 \cup T(1,1,1))^c] = \{(P_4 \cup K_{1,4} \cup P_2 \cup C_3)^c, (P_4 \cup K_{1,4} \cup P_5)^c, (P_4 \cup I_6 \cup K_1 \cup C_3)^c$, $(P_2 \cup I_8 \cup T(1,1,1))^c$, $(P_2 \cup I_8 \cup K_1 \cup C_3)^c, (I_8 \cup K_1 \cup P_5)^c, (P_4 \cup I_9 \cup K_1)^c\}$;

(3) *If* $n=3$, $[(P_4 \cup I_6 \cup T(1,1,3))^c] = \{(P_4 \cup K_{1,4} \cup P_2 \cup C_5)^c, (P_2 \cup K_{1,4} \cup P_9)^c$, $(P_4 \cup I_6 \cup K_1 \cup C_5)^c, (I_6 \cup K_1 \cup P_9)^c$, $(P_2 \cup I_8 \cup T(1,1,3))^c, (P_2 \cup I_8 \cup K_1 \cup C_5)^c, (P_2 \cup I_{13} \cup K_1)^c$, $(P_4 \cup K_1 \cup D_{3,2} \cup T(1,1,3))^c$, $(P_4 \cup 2K_1 \cup D_{3,2} \cup C_5)^c, (2K_1 \cup D_{3,2} \cup P_9)^c$, $(P_4 \cup K_1 \cup D_{4,1} \cup T(1,1,3))^c, (P_4 \cup 2K_1 \cup D_{4,1} \cup C_5)^c, (2K_1 \cup D_{4,1} \cup P_9)^c\}$;

(4) *If* $n=4$, $[(P_4 \cup I_6 \cup T(1,1,4))^c] = \{(T(1,2,5) \cup K_1 \cup T(1,1,4))^c, (T(1,2,5) \cup 2K_1 \cup C_6)^c, (T(1,2,5) \cup K_1 \cup P_3 \cup D_{3,1})^c$, $(P_4 \cup I_6 \cup K_1 \cup C_6)^c, (P_4 \cup P_3 \cup I_6 \cup D_{3,1})^c, (P_2 \cup I_8 \cup T(1,1,4))^c, (P_2 \cup I_8 \cup K_1 \cup C_6)^c, (P_2 \cup I_8 \cup P_3 \cup D_{3,1})^c$, $(P_3 \cup I_8 \cup T(1,2,2))^c, (P_4 \cup K_1 \cup D_{3,2} \cup T(1,1,4))^c, (P_4 \cup 2K_1 \cup D_{3,2} \cup C_6)^c, (P_4 \cup P_3 \cup K_1 \cup D_{3,2} \cup D_{3,1})^c$, $(P_4 \cup K_1 \cup D_{4,1} \cup T(1,1,4))^c$, $(P_4 \cup 2K_1 \cup D_{4,1} \cup C_6)^c, (P_4 \cup P_3 \cup K_1 \cup D_{4,1} \cup D_{3,1})^c\}$;

(5) *If* $n=7$, $[(P_4 \cup I_6 \cup T(1,1,7))^c] = \{(T(1,2,5) \cup K_1 \cup T(1,1,7))^c, (T(1,2,5) \cup 2K_1 \cup C_9)^c, (T(1,2,5) \cup K_1 \cup C_3 \cup T(1,2,3))^c$, $(P_4 \cup I_6 \cup K_1 \cup C_9)^c, (P_4 \cup I_6 \cup C_3 \cup T(1,2,3))^c, (P_2 \cup I_8 \cup T(1,1,7))^c, (P_2 \cup I_8 \cup K_1 \cup C_9)^c$, $(P_2 \cup I_8 \cup C_3 \cup T(1,2,3))^c$, $(P_5 \cup I_8 \cup T(1,2,3))^c$, $(K_1 \cup P_4 \cup D_{3,2} \cup T(1,1,7))^c$, $(2K_1 \cup P_4 \cup D_{3,2} \cup C_9)^c, (K_1 \cup P_4 \cup D_{3,2} \cup C_3 \cup T(1,2,3))^c$, $(K_1 \cup P_4 \cup D_{4,1} \cup T(1,1,7))^c$, $(2K_1 \cup P_4 \cup D_{4,1} \cup C_9)^c, (K_1 \cup P_4 \cup D_{4,1} \cup C_3 \cup T(1,2,3))^c\}$;

(6) *If* $n=13$, $[(P_4 \cup I_6 \cup T(1,1,13))^c] = \{(K_1 \cup T(1,2,5) \cup T(1,1,13))^c, (2K_1 \cup T(1,2,5) \cup C_{15})^c, (K_1 \cup T(1,2,5) \cup C_3 \cup C_5 \cup T(1,2,4))^c$, $(P_4 \cup I_6 \cup K_1 \cup C_{15})^c, (P_4 \cup I_6 \cup C_3 \cup C_5 \cup T(1,2,4))^c, (C_3 \cup P_9 \cup I_6 \cup T(1,2,4))^c$, $(P_2 \cup I_8 \cup T(1,1,13))^c$, $(P_2 \cup I_8 \cup K_1 \cup C_{15})^c$, $(P_2 \cup I_8 \cup C_3 \cup C_5 \cup T(1,2,4))^c, (P_5 \cup I_8 \cup C_5 \cup T(1,2,4))^c$, $(P_2 \cup I_{13} \cup C_3 \cup T(1,2,4))^c, (P_5 \cup I_{13} \cup T(1,2,4))^c$, $(K_1 \cup P_4 \cup D_{3,2} \cup T(1,1,13))^c, (2K_1 \cup P_4 \cup D_{3,2} \cup C_{15})^c$, $(K_1 \cup P_4 \cup D_{3,2} \cup C_3 \cup C_5 \cup T(1,2,4))^c$, $(K_1 \cup C_3 \cup P_9 \cup D_{3,2} \cup T(1,2,4))^c$, $(K_1 \cup P_4 \cup D_{4,1} \cup T(1,1,13))^c, (2K_1 \cup P_4 \cup D_{4,1} \cup C_{15})^c$, $(K_1 \cup P_4 \cup D_{4,1} \cup C_3 \cup C_5 \cup T(1,2,4))^c$, $(K_1 \cup C_3 \cup P_9 \cup D_{4,1} \cup T(1,2,4))^c\}$.

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